Names | Ime Ime- Essien
Dept: | Chemical Engineering
Matric: | 16/ENG01/011
Course | Mathematics (ENG 282)

Assignment 3.

1. A mathematical model is a description of a system using mathematical concepts and language. Therefore, modelling is the process of setting up a model, solving it mathematically and interpreting the result in physical and clear terms.

bi Exponential growth / decay (use of ODE)
(ii) Mixing problems.

2 $v = (t^2 + 3t)\hat{i} - 2\sin 3t\hat{j} + 3e^{2t}\hat{k}$

i $\frac{dr}{dt} = (2t+3)\hat{i} - 6\cos 3t\hat{j} + 6e^{2t}\hat{k}$

ii $\frac{d^3r}{dt^2} = 2\hat{i} + 18\sin 3t\hat{j} + 12e^{2t}\hat{k}$

iii $\left.\frac{d^3r}{dt^2}\right|_{t=0} = 2\hat{i} + 12\hat{k}$

$\left|\frac{d^2r}{dt^2}\right| = \sqrt{2^2 + 12^2} = \sqrt{148} = 2\sqrt{37} = 12.17$

3 $A = x^2 y \hat{i} + (xy + yz)\hat{j} + xz^2\hat{k}$

$B = yz\hat{i} - 3xz\hat{j} + 2xy\hat{k}$

$\phi = 3x^2 y + xyz - 4y^2 z^2 - 3$

(i) $\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$

$\frac{\partial\phi}{\partial x} = 6xy + yz$; $\frac{\partial\phi}{\partial y} = 3x^2 + xz - 8yz^2$

$\frac{\partial\phi}{\partial z} = xy - 8y^2 z$

At $(\overset{x}{1}, \overset{y}{2}, \overset{z}{1})$

$\frac{\partial\phi}{\partial x} = 6(1)(2) + (2)(1) = 12 + 2 = 14$

$\frac{\partial\phi}{\partial y} = 3(1)^2 + (1)(1) - 3(2)(1)^2 = -12$

$\frac{\partial\phi}{\partial z} = (1)(2) - 8(2)^2(1) = 2 - 32 = -30$

$\nabla\phi = 14\hat{i} - 12\hat{j} - 30\hat{k}$

(ii) $\nabla \cdot A = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$A = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$

$\nabla \cdot A = 2xy + (x+z) + 2xz$

At $(1, 2, 1)$

$\nabla \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$

$= 4 + 2 + 2 = 8$

(iii) $\nabla \times B$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$$

$= \hat{i}(2x + 3x) - \hat{j}(2y - y) + \hat{k}(-3z - z)$

$= 5x\hat{i} - y\hat{j} - 4z\hat{k}$

At $(1, 2, 1)$

$\nabla \times B = 5\hat{i} - 2\hat{j} - 4\hat{k}$

(iv) ~~gradient A~~ grad div A

grad $(2xy + (x+z) + 2xz)$

Let div $A = C = \nabla A$

$\nabla(\nabla A) = \nabla C = \hat{i}\dfrac{\partial C}{\partial x} + \hat{j}\dfrac{\partial C}{\partial y} + \hat{k}\dfrac{\partial C}{\partial z}$

$= \hat{i}(2y + 1 + 2z) + \hat{j}(2x) + \hat{k}(1 + 2x)$

At $(1, 2, 1)$

$\nabla C = \hat{i}\,(2(2)+1+2(1)) + \hat{j}\,(2(1)) + \hat{k}\,(1+(2)(1))$

$= \hat{i}\,(4+1+2) + \hat{j}\,(2) + \hat{k}\,(1+2)$

$= 7\hat{i} + 2\hat{j} + 3\hat{k}$

v) Curl Curl A

Curl $A = \nabla \times A$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 y & (xy+yz) & xz^2 \end{vmatrix}$$

$= \hat{i}\,(0-y) - \hat{j}\,(z^2-0) + \hat{k}\,(y-x^2)$

$= -y\hat{i} - z^2\hat{j} + \hat{k}\,(y-x^2)$

At $(1,2,1)$

Curl $A = -2\hat{i} - \hat{j} + \hat{k}$

Curl curl $A = \nabla \times (\nabla \times A)$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$= \hat{i}\,(1+2z) - \hat{j}\,(-2x^2-0) + \hat{k}\,(0+1)$

$= \hat{i}\,(1+2z) + 2x^2\hat{j} + \hat{k}$

At points $(1,2,1)$

$\nabla \times (\nabla \times A) = \hat{i}\,(1+2(1)) + 2(1)^2\hat{j} + \hat{k}$

$= 3\hat{i} + 2\hat{j} + \hat{k}$